Overview

We present our strong lens model of the Frontier Fields cluster MACS J0416.1-2403 and files necessary for the creation of magnification maps and errors covering the entire HST ACS field of view (202′′ × 202′′). We document the methods used in creating the model, the model inputs, and the best-fit model parameters. The deliverables include high-resolution magnification maps for $z = 1, 2, 4,$ and 9, two-dimensional deflection angles, convergence, and shear maps, along with lower-resolution convergence and shear maps from a simulation of 100 models periodically selected from the MCMC chain which can be used to derive magnification maps and errors for any source plane redshift.

We assume a flat $\Lambda$CDM cosmology, with $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$, and $H_0 = 70 \text{ km s}^{-1}$, where 1′′ = 5.34 kpc at the cluster redshift $z = 0.396$.

Lens Model

Our lens modeling method is parametric in nature. We use the publicly-available software LENSTOOL (Jullo et al. 2007), which utilizes a Markov Chain Monte Carlo to find the best-fit parameters for each mass component and arc redshift weighted by Bayesian evidence. We search the parameter space for the best-fit solution in an iterative process. We start by placing masses typical of most clusters near the center of the light distribution, and then build up the model in complexity with each iteration by adding more image constraints and more mass components. The early iterations are done under source plane optimization, where the rms scatter used to rank models is computed when the images are traced back to the source plane. The source plane optimizations serve as an adequate starting point to the final model presented here, which was computed under image plane optimization, where the rms scatter is computed for each image by tracing through the lens back to the source plane.
and back out to the image plane. The latter method is more computationally intensive, andthus is not carried out until the final iteration.

We use the “confident” arcs identified by Zitrin et al. (2013) as modeling constraints,which includes 34 images from 13 unique sources. We fix the redshift of arc system #1 to thespectroscopic redshift provided by Zitrin et al. (2013) and systems #2, 3, 4, 10, 13, 14, and16 to the spectroscopic redshifts obtained under VLT program 186.A-0798 (Balestra et al.2013, in prep.). The remaining arcs redshifts are included as free parameters in the model, byusing the range of photometric redshifts computed by Zitrin et al. (2013) as initial Bayesianpriors; however, we allow for the optimization to extend beyond this range if needed in orderertosolve for each redshift. A list of the arcs and their redshifts (spectroscopic, photometric,and model-derived with priors) is given in Table 1.

Our lens model includes two cluster-scale halos and halos assigned to each individualred sequence cluster member galaxy, all represented by pseudo-isothermal elliptical massdistributions (PIEMD; Limousin et al. 2005). The PIEMD is parameterized by a two-dimensional location in the lens plane, a lens plane redshift, ellipticity and position angle, afiducial velocity dispersion, and core radius and cut radius. The galaxies used in the modelare selected by color; those falling on the red sequence at the cluster redshift are consideredcluster members. The PIEMD halo for each galaxy is scaled by light in the ACS F775Wband according to the relationships in Limousin et al. (2005),

\[
\sigma_0 = \sigma_0^* \left( \frac{L}{L^*} \right)^{1/2} \\
r_{\text{core}} = r_{\text{core}}^* \left( \frac{L}{L^*} \right)^{1/4} \\
r_{\text{cut}} = r_{\text{cut}}^* \left( \frac{L}{L^*} \right)^{1/4}.
\]

We find that an \( L^* \) galaxy at \( z = 0.396 \) has an ACS F775W magnitude of 19.33 and set\( \sigma_0^* = 120 \text{ km s}^{-1}, \ r_{\text{core}}^* = 0.15 \text{ kpc}, \) and \( r_{\text{cut}}^* = 30 \text{ kpc}, \) following Limousin et al. (2008).These parameters are not well-constrained by the lens model, since these halos have small,localized effects on the lensing potential. We allow only three cluster member galaxies tobe optimized using their light-scaled PIEMD parameter values as priors. We also include agalaxy-like PIEMD halo centered on the bright, foreground galaxy in the south of the cluster(\( \alpha=4:16:06.817, \ \delta=-24:05:08.44 \)). The redshift of this galaxy is unknown, but can still beincluded in the model because of the degeneracy between lens mass and lensing geometry.We can solve for this galaxy’s contribution to the deflection at any redshift (however, its
model-derived mass will change), and for simplicity, place the galaxy at the cluster redshift and restrict the lensing mass to a single redshift plane. Our best-fit model includes 42 constraints and 21 free parameters, and produces an image plane rms scatter of 0.43″. The model details are included in Table 2.

**Deliverables**

We have computed magnification maps for source redshift planes of $z = 1, 2, 4, 9$. We also include the $x$ and $y$ deflection maps, representing the deflection angle from image plane to source plane in units of arcseconds, and the convergence $\kappa$ and shear $\gamma$ maps, all computed at a redshift of $z = 9$. These additional maps can be used to calculate the magnifications at any source plane redshift. We also include lower-resolution convergence and shear maps for 100 models selected at equal intervals throughout the MCMC chain, which can be used to derive the magnification errors at any source redshift. A summary of the model files is given in Table 3.

Each high-resolution map is a square $202.08'' \times 202.08''$ grid oriented north up and east to the left, with a pixel scale of 0.03″. The lower-resolution maps are set to a pixel scale of 0.12″. The FITS WCS astrometry has been matched to the preliminary HST CLASH science products created by Anton Koekemoer.¹

**Modeling strengths and caveats**

Our model best describes the magnifications in the strong lensing regime, or the region more or less enclosed by the strongly-lensed arcs (for MACS J0416.1-2403 the largest image separation is $\sim 90''$). Therefore, the objects within a few arcseconds of any images used as constraints in our model will have the most precise magnifications, especially if those images had fixed spectroscopic redshifts. The regions of the map that are most vulnerable to both systematic and modeling error are near the critical curve and far from any image constraints.

We do not recommend the use of our model for computing the magnifications in the “blank” parallel field. Extrapolating our model out to this field predicts $\mu < 1.1$ (see Figure 1), which is expected if the field is truly blank, or unmagnified ($\mu = 1$). We do not account for any mass outside of the combined FOV of existing HST data (additional

¹e.g. http://archive.stsci.edu/pub/hlsp/frontier/internal/macs0416/images/hst/clash/hlsp_clash_hst_acs-30mas_macs0416_f606w_v1_drz.fits
cluster members, large-scale structure, etc.) which could boost the lensing in this region, thus, this extrapolation results in a crude, unconstrained estimate of the magnification. For the parallel field, we suggest that one uses maps generated by other mapping techniques, which include weak lensing as constraints.

**Future model revisions**

The magnification maps we have created in this model version of MACS J0416.1-2403 are at this moment very well constrained and provide precise values for the magnification in the strong lensing regime. However, no model is exact and can always be improved upon. New arc identifications can add constraint to new regions of the map and spectroscopic redshifts of arcs can greatly reduce magnification errors in areas near those images. We will use our current models to identify new image systems and include them in new model version and will update known image systems with new spectroscopic redshifts as they become available.

**REFERENCES**


This preprint was prepared with the AAS l\TeX\ macros v5.2.
Fig. 1.— Magnification maps centered on the HST Frontier Field cluster MACS J0416.1-2403 (upper right) and parallel field for a source redshift of $z = 9$. Overlaid are the CCD chip boundaries of the ACS (blue) and WFC3/IR (red) for both Frontier Field pointings. Magnification contour levels are $\mu = 1.1, 1.2, 1.5, 2, 5, 10$. 
#### Table 1. Modeling constraints

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<th>Dec.</th>
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<th>Photo z$^c$</th>
<th>Model z</th>
<th>z prior$^d$</th>
<th>Image plane rms (′′)</th>
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<td>···</td>
<td>···</td>
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Note. — Arc identifications by Zitrin et al. (2013).

$^a$Spectroscopic redshift reported by Zitrin et al. (2013).

$^b$Spectroscopic redshifts from Grillo et al. (2013, in prep.), obtained from VLT program 186.A-0798 (Balestra et al. 2013, in prep.).

$^c$Range of photometric redshifts reported by Zitrin et al. (2013).

$^d$Bayesian priors on redshift are for the final model iteration.
Table 2. Details of Len Model

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<th>Component</th>
<th>∆RA (′′)</th>
<th>∆Dec (′′)</th>
<th>e</th>
<th>θ (°)</th>
<th>r_{core} (kpc)</th>
<th>r_{cut} (kpc)</th>
<th>σ (km s^{-1})</th>
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<td>17.2^{+0.4}_{-0.7}</td>
<td>15.3^{+0.6}_{-0.4}</td>
<td>0.45^{+0.05}_{-0.04}</td>
<td>-36.8^{+1.3}_{-0.8}</td>
<td>115^{+12}_{-10}</td>
<td>[1500] 1080^{+41}_{-47}</td>
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<tr>
<td>halo #2</td>
<td>-21.2 ± 0.7</td>
<td>-43.1^{+1.9}_{-1.2}</td>
<td>0.06^{+0.11}_{-0.06}</td>
<td>-44.7 ± 4</td>
<td>90.8^{+7}_{-10}</td>
<td>[1500] 849^{+30}_{-50}</td>
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<td>[-50.7]</td>
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<td>[69.9] 99.6^{+42}_{-14}</td>
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<td>[0.182]</td>
<td>[36.5] 167 ± 55</td>
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<td>[0.000]</td>
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<td>[0.150]</td>
<td>[50.0] 42.6^{+12.6}_{-8.1}</td>
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<td>⋯</td>
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<td>⋯</td>
<td>[0.15]</td>
<td>[30] 120</td>
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*Model-derived velocity dispersion at a lens plane redshift of z = 0.396.

Note. — Parameters for best-fit model and 1σ errors. Values in brackets are not optimized, or “frozen” parameters. ∆RA and ∆Dec are measured with respect to the galaxy at α=4:16:08.331, δ=-24:04:17.74, position angles are measured north of west.

Table 3. List of Delivered Files

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<th>File name</th>
<th>Deliverable</th>
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<th>Size [pixels]</th>
<th>Pixel scale [′′]</th>
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Note. — All high-resolution FITS files are named with the same format: hlsp_frontier_model_macsj0416_sharon_[column 1]_v1.fits.