

Scattered Light in Short Wavelength Low Dispersion Spectra

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I briefly discuss and add to a more extended analysis of the cross-dispersion grating scattering profile and its effect on SWP-LO spectra. The initial interest in this problem arose due to the contamination of spectra for stars later than B by light scattered from the LW range onto weak continuum regions in the SW range. A more complete report has been published by Basri, Clarke, and Haisch (1985, hereafter BCH). We describe in that paper how we arrived at the scattering profile; I discuss here only its main effects and how to analyse and remove them from observations. Data has also been pointed out since that paper which requires modification of the scattering profile used in it, though its conclusions remain intact.

Any grating has the property that it scatters light of a given wavelength to other angles along the dispersion; the characteristic shape of this scattering profile varies with each grating, as does the total amount of power removed from a monochromatic (central) flux bin. In general, this total power removed will increase as the inverse square of the wavelength and the shape of the profile may change as well. In IUE low dispersion spectra, it is the cross-dispersion grating whose scattering is of interest. The flight gratings themselves were not measured, but gratings from the same master were studied by Mount and Fastie (1978, hereafter MF). This scattering is important primarily because the SWP camera photocathode is sensitive to light at wavelengths into the visible range. Consequently, although light longward of 2000Å does not appear directly on the camera, some of that light is scattered onto it and is detected. This is most obvious in very well exposed spectra of cool stars (which are quite bright longward of 2000Å relative to shortward of it) where an appreciable signal is detected across the entire camera, even at positions corresponding to wavelengths to which the camera is blind. The scattering profile decreases rapidly from the central wavelength, but low level wings obviously persist for hundreds of angstroms.

Previous partial analyses of this problem have appeared in Clarke (1981) and Crivellari and Praderie (1982). These authors approximated the far wings by a flat "pedestal" of signal, and the latter authors estimated the shape of the scattered light by comparison of observed spectra with theoretical models. BCH used a synthetic scattering model to calculate explicitly the effect of different scattering profiles on computed spectra, then found the scattering profile which best fit selected IUE observations which included well-observed scattered light. They began with the MF scattering measurements and extended these both far out in the wings and all the way

to the central wavelength. They also explored certain modifications to the MF profile.

After publication of BCH, Feldman (1985) noticed that the derived scattering profile they used was incompatible with extremely overexposed cometary Lyman-alpha spectra he had in the region within 100A of line center. BCH had not actually derived a profile there from observations, but had taken a profile consistent with the measurements of MF. As Feldman discusses, those measurements cannot be an accurate reflection of the core of the scattering profile of the flight grating. I have rederived a scattering profile based on Feldman's data for this Newsletter. His observations are shown in Figure 1, with the curve I fit through them to yield the new scattering profile core. What is shown is the excess flux in a highly overexposed observation compared to a companion well-exposed observation. In Figure 2, I show the old and new scattering profile cores. The main effect of this difference is to require that much less light is scattered from the central wavelength than was required to feed the high inner wings of the MF profile. The amount of central power scattered out at the fiducial wavelength of BCH is reduced from 4% to 0.8% (which translates to a 3% loss at Lyman-alpha instead of 16%). The far wings of the scattering profile must still reproduce the Canopus analysis in BCH, and so are almost the same.

Close attention must be paid to the form in which the flux is used. Energy units are not appropriate because the light is scattered as individual photons, and the signal appearing at a given position along the dispersion is composed of photons of various energies. The signal they produce depends on the photocathode response to the energy of each photon (whether scattered or not). The most appropriate form in which to deal with the spectra is therefore flux numbers, and one must be prepared to convert these to photons and vice versa. Fortunately, the photocathode response is slowly varying over the region of interest. More importantly, a given photon which produces a certain flux number at the appropriate place in the spectrum will produce the same flux number if scattered to a different location in the dispersion direction (assuming proper flat fielding). Thus a scattering analysis carried out entirely with flux numbers avoids dealing with the photocathode response function.

In order to compute the scattering, a normalized scattering profile which extends over the full computational wavelength range must be found for each wavelength. The normalization must account for the varying amount of flux retained in the central wavelength as discussed in BCH; this has the effect of raising the wings as the central power decreases. In the scattering calculation, the center of this profile is registered successively at each wavelength in the spectrum, renormalized, then the appropriate fraction of the flux incident at that wavelength is added into each wavelength

of an accumulating scattered spectrum. This procedure can be employed to take a known incident spectrum and discover the effect of the grating scattering on it. One can use flux number spectra from GO tapes or derive them for theoretical models from the known instrumental response. Of primary interest for the scattering calculation is the spectrum in the LW range from 2000-3000A, as most of the light is generally scattered from there. If the intrinsic spectrum is known somehow, subroutine SCAT in the accompanying FORTRAN code can be used to derive a scattered spectrum to compare with observations. It is not necessary to have all the details of the LW spectrum exactly right (since they are very washed together in the scattering process); a generically appropriate spectrum there should be adequate in most applications.

The more usual situation is that one has an observed spectrum including scattered light and would like to know the intrinsic spectrum. With knowledge of the actual scattering profile, the procedure can be run more or less in reverse (apart from some subtleties discussed by BCH). A prescription for doing so is provided in subroutine DESCAT. In Figure 3 is displayed the results of grating scattering on a synthetic spectrum produced by steepening the known intrinsic solar spectrum to mimic a very late-type star (other results are shown in BCH). I caution that a real late-type star will not actually look like this, since the actual UV continuum will depend on the strength of its particular chromospheric contribution. The Figure is illustrative of the sort of effects that grating scattering can have on both the continuum and emission lines.

As the slope of the continuum becomes shallower, the contribution of scattered light becomes smaller. Thus, for a solar-type spectrum, scattered light is only really important for the shortward end of the SW range. Earlier-type spectra paradoxically may contain visible scattered light more often, due to the exposure times imposed by the limited dynamic range of the IUE cameras and the fact that the bright part of the stellar continuum starts to move into the SW range.

I conclude by summarizing the effects of grating scattering on IUE spectra. For relatively flat spectra with relatively small (<5x) contrast in narrow features, there is essentially no effect because light removed at each wavelength is replaced from adjacent wavelengths. As the continuum slope steepens, there comes a point at which the scattered light even from several hundred angstroms away is larger than the intrinsic continuum. The effect of the scattered light can then vary from flattening the continuum a bit to becoming essentially the entire (flat) signal. In A-F stars the effect is usually small longward of the observed steep drop in the continuum, and then becomes very large quite rapidly. In solar-type stars the continuum begins to be flattened below 1700A, and this flattening occurs at longer wavelengths for later type

stars. The effects can show up for increasingly shallow intrinsic SW continuum slopes in later type stars because the contrast between the LW and SW regions becomes increasingly large. Strong narrow absorption features will have their central intensities increased proportional to their intrinsic depth for all spectral types. Similarly, the apparent contrast of emission lines with the continuum is reduced to the extent that the observed continuum is composed of scattered light near the emission line.

References

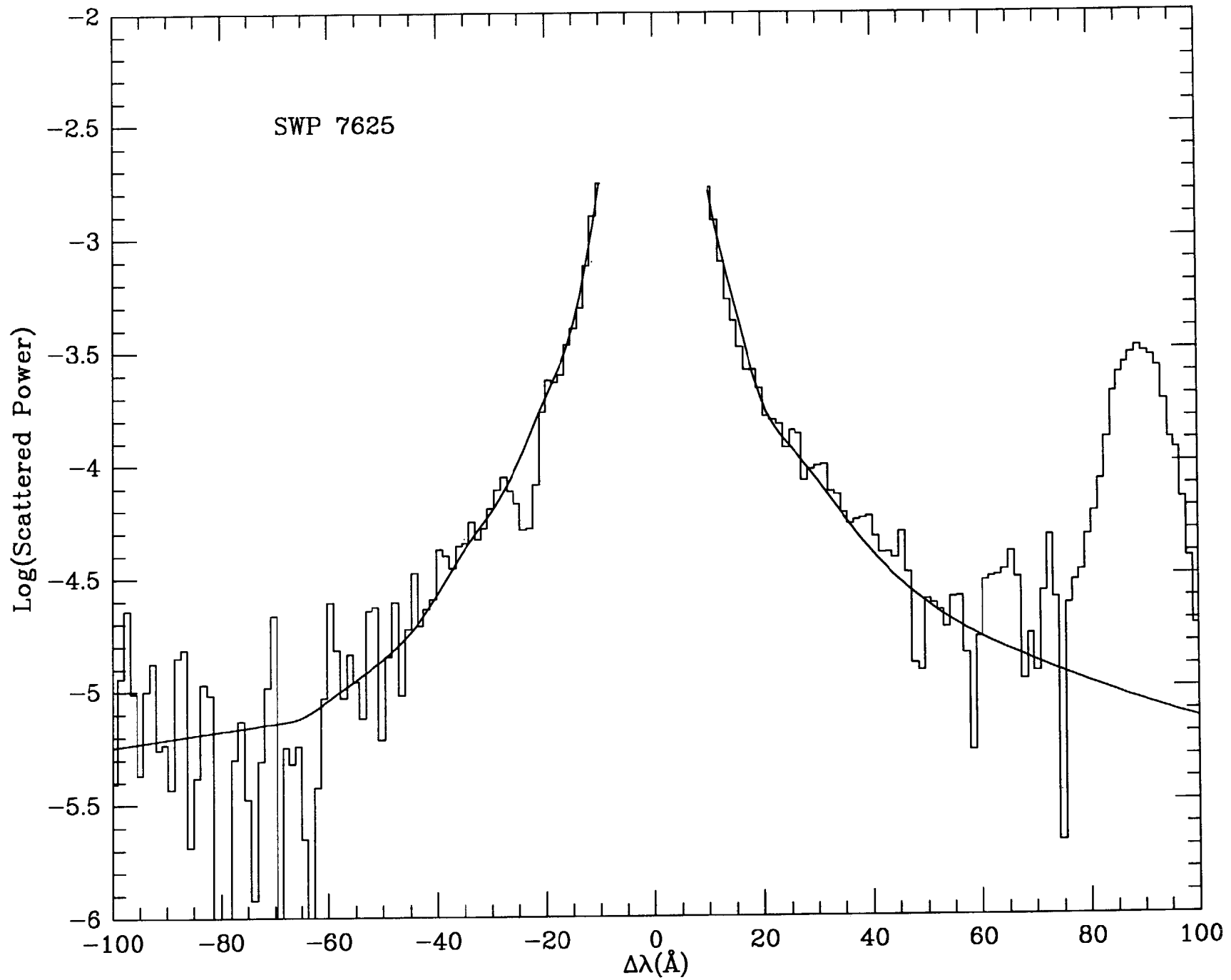
Basri, G., Clarke, J.T., and Haisch, B.M.: 1985, *Astronomy and Astrophysics*, **144**, 161.

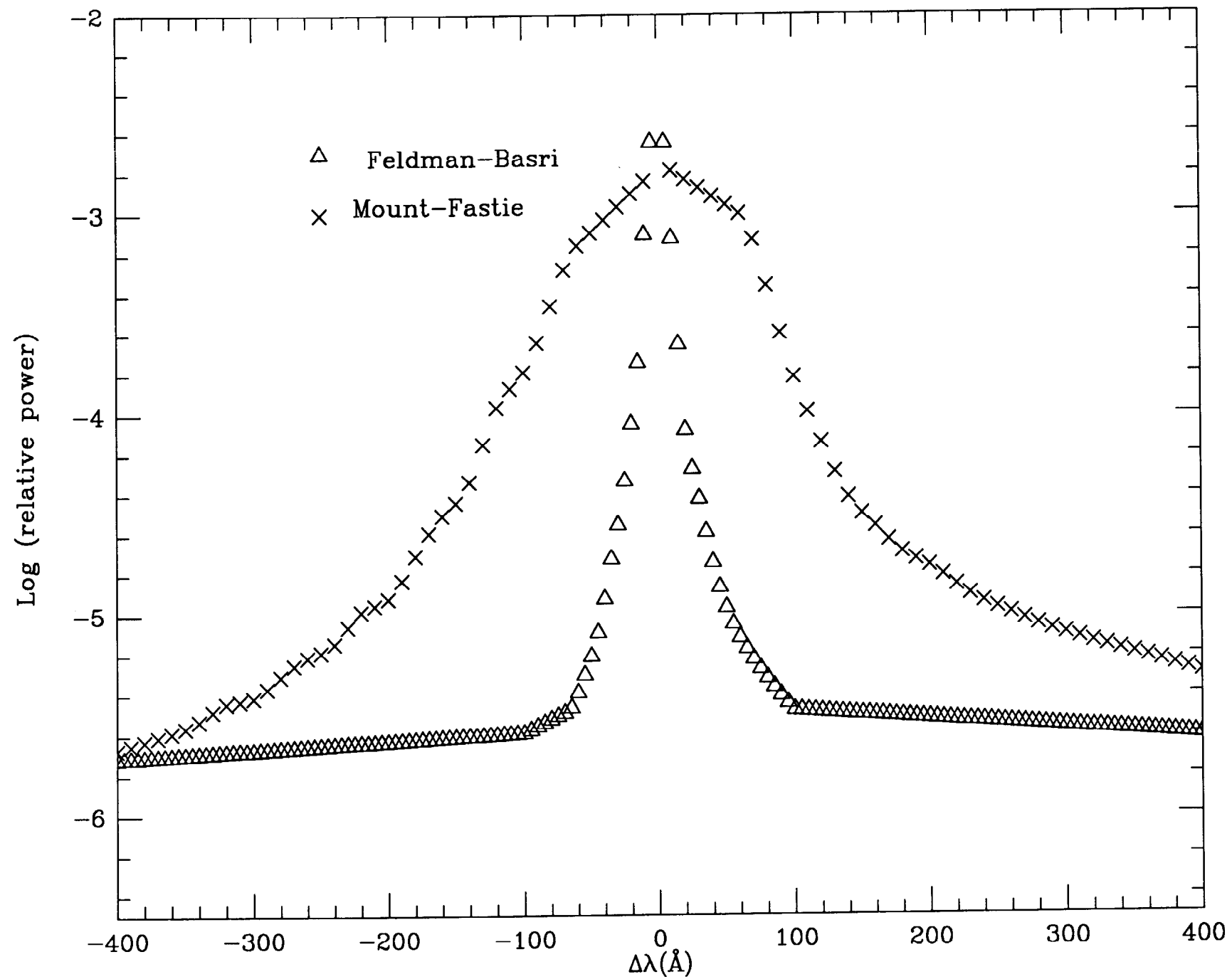
Clarke, J.T.: 1981, *NASA IUE Newsletter* #14, 143.

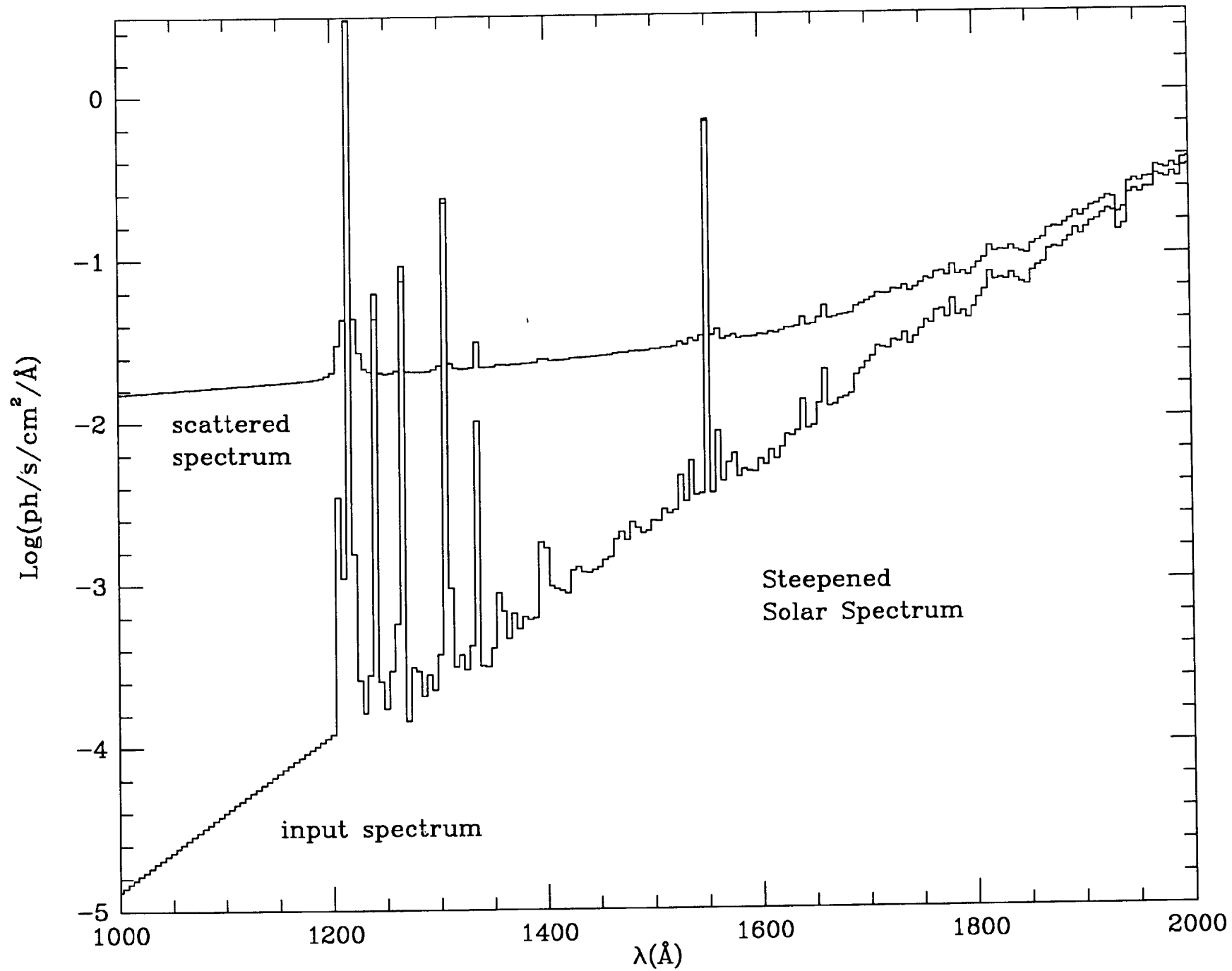
Crivellari, L. and Praderie, F.: 1982, *Astronomy and Astrophysics* **107**, 75.

Feldman, P.: 1985, in preparation.

Mount, G.H. and Fastie, W.G.: 1978, *Applied Optics* **17**, 3108.







```

program iuescat
c Grating scattering program for IUE described by Basri, Clarke and
c Haisch 1985, Astronomy and Astrophysics, v.144, p.161.
c Modified for Feldman scattering profile, May, 1985
  dimension ar(500),tab(5,500),w(500),wt(41),st(41)
  dimension sct(1001),spl(500),sp2(500),scl(500),cw(500)
  character*20 fn

  data wt/-.1.0E+02,-9.5E+01,-9.0E+01,-8.5E+01,-8.0E+01,-7.5E+01
&,-7.0E+01,-6.5E+01,-6.0E+01,-5.5E+01,-5.0E+01,-4.5E+01
&,-4.0E+01,-3.5E+01,-3.0E+01,-2.5E+01,-2.0E+01,-1.5E+01
&,-1.0E+01,-5.0E+00, 0.0E+00, 5.0E+00, 1.0E+01, 1.5E+01
&, 2.0E+01, 2.5E+01, 3.0E+01, 3.5E+01, 4.0E+01, 4.5E+01
&, 5.0E+01, 5.5E+01, 6.0E+01, 6.5E+01, 7.0E+01, 7.5E+01
&, 8.0E+01, 8.5E+01, 9.0E+01, 9.5E+01, 1.0E+02/

  data st/-5.245,-5.228,-5.211,-5.194,-5.177,-5.160
&,-5.143,-5.119,-5.042,-4.955,-4.863,-4.746
&,-4.576,-4.379,-4.208,-3.988,-3.703,-3.396
&,-2.761,-1.922,-1.500,-1.986,-2.779,-3.307
&,-3.735,-3.927,-4.079,-4.244,-4.395,-4.520
&,-4.622,-4.704,-4.771,-4.829,-4.881,-4.929
&,-4.975,-5.019,-5.061,-5.100,-5.135/

  llt=41
  na=2
c read in spectrum to scatter
  type *, 'length(14), input spectrum file name'
  accept 10, lent, fn
  format(14, a20)
  open(1, file=fn, readonly, form='unformatted', err=20, status='old')
  call readf(lent, na, ar, tab)
  close(1)

  do i=1, lent
  cw(i)=tab(1, i)
  sp2(i)=tab(2, i)
  scl(i)=0.
  end do

c interpolate spectrum onto 5A bins
  len=(cw(lent)-cw(1))/5.
  w(1)=cw(1)
  spl(1)=sp2(1)
  do i=2, len
  w(i)=w(i-1)+5.
  end do
  call intep(w(2), spl(2), cw, sp2, lent, ier)
  do i=3, len
  call sintep(w(i), spl(i), cw, sp2, lent, ier)
  end do

c convert spectrum to flux numbers (sort of) if given in energy units
c this does not take account of quantum efficiency variations
  if(spl(len/2).lt.1.e-7) then
  do i=1, len
  spl(i)=spl(i)*w(i)/(1.986e-8)
  end do

  end if

  type *, 'scatter (1) or de-scatter (0)'
  accept *, isct
  cp=0.992

```

```

ef=1000
pd=-2.3
dw=wt(2)-wt(1) !assume equal wavelength bins

c fill actual scattering profile from template
  st(20)=pd
  st(22)=pd
  lsc=2*len+1
  ic=lsc/2-llt/2
  do n=1, llt
  sct(n+ic)=st(n)
  end do

c and add exponential wings
  np=ic+1
  fex=.43429
  i=lsc-np+1
  do n=1, np-1
  sct(np-n)=sct(np)-fex*n*dw/ef
  sct(i+n)=sct(i)-fex*n*dw/ef
  end do

c prepare final scattering profile
  do n=1, lsc
  sct(n)=10**(sct(n))
  end do
  ic=lsc/2+1
  sct(ic)=0.
  sum=0.

c normalization of scattering profile at midspectrum
c conserves light over +/- len*dw angstroms
  do i=1, lsc
  sum=sum+sct(i)
  end do
  do i=1, lsc
  sct(i)=sct(i)*(1.-cp)/sum
  end do

  if(isct.eq.1) call scat(len, w, sct, spl, scl, cp, ic)
  if(isct.eq.0) call descats(len, w, sct, spl, sp2, cp, ic)

c output from calculation
  type *, 'save this experiment? (0=no)'
  accept *, i
  if(i.eq.0) stop
  type *, 'save file name'
  read 5, fn
  format(a20)
c do whatever you want here

  stop
  type *, 'error opening file'
  end

```



```

subroutine scat(len,w,sct,sfn,sc,cp,ic)
dimension w(1),sct(1),sfn(1),sc(1)
dimension s(500)
c input spectrum sfn of length len assumed in flux numbers
c with wavelength scale w
c scattered spectrum is sc (flux numbers)
c the template scattering profile sct has central power cp
c at ic and length 2*len+1, and totals to 1.0

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```

type *, 'scattering calculation'
wh=(w(len)+w(1))/2.
type *, 'central fiducial wavelength',wh
sum=1.-cp

```

```

do n=1,len
sq=(wh/w(n))**2
c register scat. prof. to incident wavel.
sm=0.

```

```

do i=1,len
s(i)=sct(ic-n+1)*sq
sm=sm+s(i)
end do

```

```

s(n)=1.-sq*sum
c add scattered power from this wavel. to scat. spec.
do i=1,len
sc(i)=sc(i)+s(i)*sfn(n)
end do

```

```

end do

```

```

return
end

```

```

subroutine descatscat(len,w,sct,sinp,sc,cp,ic)
dimension w(1),sct(1),sinp(1),sc(1)
dimension s(500),sfn(500)

```

```

c input spectrum sinp assumed to be in flux numbers

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```

wh=(w(len)+w(1))/2.
sum=1.-cp
do n=1,len
sfn(n)=sinp(n)
sc(n)=0.
end do

```

```

type *, 'de-scattering calculation'

```

```

do n=1,len
sq=(wh/w(n))**2
sfn(n)=sfn(n)/(1.-sum*sq)
sfnn=sfn(n)

```

```

c We assume all light <1150A is pure scattered light
if(w(n).lt.1150.) sfnn=sfnn*1.e-5

```

```

c register scat. prof. to incident wavel.
do i=1,len
s(i)=sct(ic-n+1)*sq
end do

```

```

s(n)=0.0
c add up scattered counts
do i=1,len
sc(i)=sc(i)+s(i)*sfnn
end do

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```

end do

```

```

c subtract back out light scattered into each pixel
do n=1,len
sc(n)=sfn(n)-sc(n)
end do
c sc now contains the de-scattered spectrum in flux #'s

return
end

```

```

subroutine readf(len,na,ar,tab)
c special purpose I/O; you probably want something else
dimension ar(len),tab(5,len)
character*20 fn
do n=1,na
read (1) ar
do i=1,len
tab(n,i)=ar(i)
end do
type *, 'read array',n,ar(1),ar(len)
end do

return
end

```

```

SUBROUTINE INTEP (XP,P,X,F,N,IER)
C
C INTERPOLATES A FUNCTION VALUE 'P' FOR A GIVEN ARGUMENT 'XP'
C USING A TABLE OF 'N' VALUES OF 'X' AND 'F'.
C 'X' MUST BE MONOTONIC (APPARENTLY - DJ.)
C EMPLOYS A SPLINE INTERPOLATION SCHEME BASED ON THE HERMITE
C POLYS. SOURCE : U.S.A.F. SURVEYS IN GEOPHYSICS #272.
C IF 'XP' IS BEYOND THE RANGE OF 'X' THE VALUE OF 'F' AT THE
C CORRESPONDING END IS ASSUMED, AND 'IER' IS SET TO 2; OTHERWISE,
C 'IER' IS SET TO 1.
C CALLING :
C FOR RANDOM VALUES OF 'XP', OR THE FIRST TIME, CALL INTEP AS
C IN THE CALLING SEQ. ABOVE. FOR MONOTONICALLY INCREASING OR
C DECREASING VALUES OF 'XP' (AFTER INTEP HAS BEEN CALLED ONCE),
C CALL EINTPEP, WITH THE SAME ARGUMENTS (SEE ENTRY POINT BELOW).
C
REAL LP1, LP2, L1, L2, F(1), X(1)
IF (N.LT.2) THEN
  P=F(1)
  RETURN
END IF
IER=1
IO=1
IUP=0
IF (X(2).LT.X(1)) IUP=1
N1=N-1
IF ((XP.GE.X(N) .AND. IUP.EQ.0) .OR.
  & (XP.LE.X(1) .AND. IUP.EQ.1)) THEN
  P=F(N)
  GO TO 6
ELSE IF ((XP.LE.X(1) .AND. IUP.EQ.0) .OR.
  & (XP.GE.X(1) .AND. IUP.EQ.1)) THEN
  P=F(1)
  IER=2
  RETURN
END IF
ENTRY EINTPEP (XP,P,X,F,N,IER)
DO I=IO,N
  & IF ((XP.LT.X(I) .AND. IUP.EQ.0) .OR.
  & (XP.GT.X(I) .AND. IUP.EQ.1)) GO TO 2
END DO
GO TO 5
2 IER=1
I=I-1
IF (I.EQ.IO-1) GO TO 4
IO=I+1
LP1=1/(X(I)-X(I+1))
LP2=-LP1
IF (I.EQ.1) THEN
  FP1=(F(2)-F(1))/(X(2)-X(1))
ELSE
  FP1=(F(I+1)-F(I-1))/(X(I+1)-X(I-1))
ENDIF
IF (I.GE.N1) THEN
  FP2=(F(N)-F(N-1))/(X(N)-X(N-1))
ELSE

```

```

FP2=(F(I+2)-F(I))/(X(I+2)-X(I))
END IF
4 XPI1=XP-X(I+1)
XPI=X(X(I))
L1=XPI*LP1
L2=XPI*LP2
& P=(F(I)*(1.-2.*LP1*XPI)+FP1*XPI)*L1*L1 +
& (F(I+1)*(1.-2.*LP2*XPI1)+FP2*XPI1)*L2*L2
RETURN
END

```

```

program luescat
c Grating scattering program for IUE described by Basri, Clarke and
c Haisch 1985, Astronomy and Astrophysics, v.144, p.161.
c Modified for Feldman scattering profile, May, 1985
  dimension ar(500),tab(5,500),w(500),wt(41),st(41)
  dimension sct(1001),sp1(500),sp2(500),scl(500),cw(500)
  character*20 fn

  data wt/-1.0E+02,-9.5E+01,-9.0E+01,-8.5E+01,-8.0E+01,-7.5E+01
&,-7.0E+01,-6.5E+01,-6.0E+01,-5.5E+01,-5.0E+01,-4.5E+01
&,-4.0E+01,-3.5E+01,-3.0E+01,-2.5E+01,-2.0E+01,-1.5E+01
&,-1.0E+01,-5.0E+00, 0.0E+00, 5.0E+00, 1.0E+01, 1.5E+01
&, 2.0E+01, 2.5E+01, 3.0E+01, 3.5E+01, 4.0E+01, 4.5E+01
&, 5.0E+01, 5.5E+01, 6.0E+01, 6.5E+01, 7.0E+01, 7.5E+01
&, 8.0E+01, 8.5E+01, 9.0E+01, 9.5E+01, 1.0E+02/

  data st/-5.245 , -5.228 , -5.211 , -5.194 , -5.177 , -5.160
&,-5.143 , -5.119 , -5.042 , -4.955 , -4.863 , -4.746
&,-4.576 , -4.379 , -4.208 , -3.988 , -3.703 , -3.396
&,-2.761 , -1.922 , -1.500 , -1.986 , -2.779 , -3.307
&,-3.735 , -3.927 , -4.079 , -4.244 , -4.395 , -4.520
&,-4.622 , -4.704 , -4.771 , -4.829 , -4.881 , -4.929
&,-4.975 , -5.019 , -5.061 , -5.100 , -5.135/

  llt=41
  na=2
c read in spectrum to scatter
  type *, 'length(14), input spectrum file name'
  accept 10, lent, fn
10  format (i4, a20)
  open(1, file=fn, readonly, form='unformatted', err=20, status='old')
  call readf(lent, na, ar, tab)
  close(1)

  do i=1, lent
  cw(i)=tab(1, i)
  sp2(i)=tab(2, i)
  scl(i)=0.
  end do

c interpolate spectrum onto 5A bins
  len=(cw(lent)-cw(1))/5.
  w(1)=cw(1)
  sp1(1)=sp2(1)
  do i=2, len
  w(i)=w(i-1)+5.
  end do
  call intep(w(2), sp1(2), cw, sp2, lent, ier)
  do i=3, len
  call eintep(w(i), sp1(i), cw, sp2, lent, ier)
  end do

c convert spectrum to flux numbers (sort of) if given in energy units
c this does not take account of quantum efficiency variations
  if(sp1(len/2).lt.1.e-7) then
    do i=1, len
    sp1(i)=sp1(i)*w(i)/(1.986e-8)
    end do
  end if

  type *, 'scatter (1) or de-scatter (0)'
  accept *, isct
  cp=0.992

```

```

ef=1000
pd=-2.3
dw=wt(2)-wt(1)           !assume equal wavelength bins

c fill actual scattering profile from template
st(20)=pd
st(22)=pd
lsc=2*len+1
ic=lsc/2-llt/2
do n=1, llt
sct(n+ic)=st(n)
end do

c and add exponential wings
np=ic+1
fex=.43429
i=lsc-np+1
do n=1, np-1
sct(np-n)=sct(np)-fex*n*dw/ef
sct(i+n)=sct(i)-fex*n*dw/ef
end do

c prepare final scattering profile
do n=1, lsc
sct(n)=10**(sct(n))
end do
ic=lsc/2+1
sct(ic)=0.
sum=0.

c normalization of scattering profile at midspectrum
c conserves light over +/- len*dw angstroms
do i=1, lsc
sum=sum+sct(i)
end do
do i=1, lsc
sct(i)=sct(i)*(1.-cp)/sum
end do

if(isct.eq.1) call scat(len,w,sct,sp1,sc1,cp,ic)
if(isct.eq.0) call descats(len,w,sct,sp1,sp2,cp,ic)

c output from calculation
type *, 'save this experiment? (0=no) '
accept *, i
if(i.eq.0) stop
type *, 'save file name'
read 5, fn
5 format(a20)
c do whatever you want here

20 stop
type *, 'error opening file'
end

```

```

        subroutine scat(len,w,sct,sfn,sc,cp,ic)
        dimension w(1),sct(1),sfn(1),sc(1)
        dimension s(500)
c   input spectrum sfn of length len assumed in flux numbers
c   with wavelength scale w
c   scattered spectrum is sc (flux numbers)
c   the template scattering profile sct has central power cp
c   at ic and length 2*len+1, and totals to 1.0

```

```

        type *, 'scattering calculation'
        wh=(w(len)+w(1))/2.
        type *, 'central fiducial wavelength',wh
        sum=1.-cp

```

```

        do n=1,len
        sq=(wh/w(n))**2
c   register scat. prof. to incident wavel.
        sm=0.
            do i=1,len
            s(i)=sct(ic-n+i)*sq
            sm=sm+s(i)
            end do
        s(n)=1.-sq*sum
c   add scattered power from this wavel. to scat. spec.
            do i=1,len
            sc(i)=sc(i)+s(i)*sfn(n)
            end do
        end do

        return
        end

```

```

        subroutine descat(len,w,sct,sinp,sc,cp,ic)
        dimension w(1),sct(1),sinp(1),sc(1)
        dimension s(500),sfn(500)
c   input spectrum sinp assumed to be in flux numbers

```

```

        wh=(w(len)+w(1))/2.
        sum=1.-cp
        do n=1,len
        sfn(n)=sinp(n)
        sc(n)=0.
        end do

```

```

        type *, 'de-scattering calculation'

```

```

        do n=1,len
        sq=(wh/w(n))**2
c   replace scattered light at each pixel (known in advance)
        sfn(n)=sfn(n)/(1.-sum*sq)
        sfnn=sfn(n)
c   We assume all light <1150A is pure scattered light
        if(w(n).lt.1150.) sfnn=sfnn*1.e-5
c   register scat. prof. to incident wavel.
            do i=1,len
            s(i)=sct(ic-n+i)*sq
            end do
        s(n)=0.0
c   add up scattered counts
            do i=1,len
            sc(i)=sc(i)+s(i)*sfnn
            end do
        end do

```

```
c subtract back out light scattered into each pixel
  do n=1,len
  sc(n)=sfn(n)-sc(n)
  end do
```

```
c sc now contains the de-scattered spectrum in flux #'s

  return
end
```

```
      subroutine readf(len,na,ar,tab)
c special purpose I/O; you probably want something else
  dimension ar(len),tab(5,len)
  character*20 fn
      do n=1,na
      read (1) ar
      do i=1,len
      tab(n,i)=ar(i)
      end do
  type *, 'read array',n,ar(1),ar(len)
  end do
  return
end
```

SUBROUTINE INTEP (XP,P,X,F,N,IER)

C INTERPOLATES A FUNCTION VALUE 'P' FOR A GIVEN ARGUMENT 'XP'
C USING A TABLE OF 'N' VALUES OF 'X' AND 'F'.
C 'X' MUST BE MONOTONIC (APPARENTLY - DJ.) .
C EMPLOYS A SPLINE INTERPOLATION SCHEME BASED ON THE HERMITE
C POLYS. SOURCE : U.S.A.F. SURVEYS IN GEOPHYSICS #272.
C IF 'XP' IS BEYOND THE RANGE OF 'X', THE VALUE OF 'F' AT THE
C CORRESPONDING END IS ASSUMED, AND 'IER' IS SET TO 2; OTHERWISE,
C 'IER' IS SET TO 1.
C CALLING :
C FOR RANDOM VALUES OF 'XP', OR THE FIRST TIME, CALL INTEP AS
C IN THE CALLING SEQ. ABOVE. FOR MONOTONICALLY INCREASING OR
C DECREASING VALUES OF 'XP' (AFTER INTEP HAS BEEN CALLED ONCE),
C CALL EINTEP, WITH THE SAME ARGUMENTS (SEE ENTRY POINT BELOW).

REAL LP1, LP2, L1, L2, F(1), X(1)

IF (N.LT.2) THEN
P=F(1)
RETURN
END IF

IER=1
IO=1
IUP=0
IF (X(2).LT.X(1)) IUP=1
N1=N-1

5 & IF ((XP.GE.X(N) .AND. IUP.EQ.0) .OR.
& (XP.LE.X(1) .AND. IUP.EQ.1)) THEN
P=F(N)
GO TO 6
& ELSE IF ((XP.LE.X(1) .AND. IUP.EQ.0) .OR.
& (XP.GE.X(1) .AND. IUP.EQ.1)) THEN
6 P=F(1)
IER=2
RETURN
END IF

ENTRY EINTEP (XP,P,X,F,N,IER)

8 DO I=IO,N
& IF ((XP.LT.X(I) .AND. IUP.EQ.0) .OR.
& (XP.GT.X(I) .AND. IUP.EQ.1)) GO TO 2
END DO
GO TO 5

2 IER=1
I=I-1
IF (I.EQ.IO-1) GO TO 4
IO=I+1
LP1=1./(X(I)-X(I+1))
LP2=-LP1

IF (I.EQ.1) THEN
FP1=(F(2)-F(1))/(X(2)-X(1))
ELSE
FP1=(F(I+1)-F(I-1))/(X(I+1)-X(I-1))
ENDIF
IF (I.GE.N1) THEN
FP2=(F(N)-F(N-1))/(X(N)-X(N-1))
ELSE

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      FP2=(F(I+2)-F(I))/(X(I+2)-X(I))  
END IF
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XPI1=XP-X(I+1)
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XPI=XP-X(I)
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L1=XPI1*LP1
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L2=XPI*LP2
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& P=(F(I)*(1.-2.*LP1*XPI)+FP1*XPI)*L1*L1 +  
    (F(I+1)*(1.-2.*LP2*XPI1)+FP2*XPI1)*L2*L2
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RETURN
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END
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