INTRODUCTION

The resolving power \( R = \lambda / \text{FWHM} \) of the IUE spectrographs varies from \( 1.0 \times 10^4 \) to \( 1.5 \times 10^4 \) over the entire wavelength range covered (Boggess, et al., 1978). Therefore the IUE velocity resolution varies from \( 20 \text{ km s}^{-1} \) to \( 30 \text{ km s}^{-1} \). If it is assumed that the centroid of a line can be determined to approximately 10% of its FWHM, then a measured radial velocity should be accurate to about \( 2 \text{ km s}^{-1} \) (best case). The orbital velocity of the Earth about the Sun \( (\sim 30 \text{ km s}^{-1}) \) and the velocity of the spacecraft about the Earth \( (\sim 4 \text{ km s}^{-1} \text{ at perigee}) \) are both larger than the best possible velocity determinations and their effect should be removed from the data.

Two subprograms, VELSUN and VELSAT, have been written for IUESIPS, the international Ultraviolet Explorer Spectral Image Processing System, to calculate orbital velocities. VELSUN determines the velocity vector of the Earth at a given time using the orbital elements of the Earth and the time derivatives of these elements (A.E., 1980). VELSAT determines the velocity vector of the spacecraft about the Earth at a given time using all the orbital elements of the spacecraft for Nov. 22, 1979, except for the period, which is set to exactly one sidereal day. Since the orbit of the spacecraft is periodically adjusted to maintain a sidereal period and, moreover, an approximately fixed ground path, it is not necessary to update the orbital elements used by the program. This program is accurate to \( \pm 0.25 \text{ km s}^{-1} \) over the entire life of the spacecraft (launch to present), and VELSUN is accurate to better than \( 0.01 \text{ km s} \). Both of these subprograms will be added to IUESIPS in the near future.
EQUATIONS USED BY VELSUN AND VELSAT

The following sets of equations giving the rectangular coordinates of a body in its orbit as a function of the orbital elements were adopted (A. D. Dubyago, 1961):

\[
\begin{align*}
    x &= r \left( \cos u \cos \Omega - \sin u \cos i \sin \Omega \right), \\
    y &= r \left( \cos u \sin \Omega + \sin u \cos i \cos \Omega \right), \\
    z &= r \sin u \sin i.
\end{align*}
\]

(1) and

\[
\mu = \frac{k \sqrt{1 + m}}{a^{3/2}} = \frac{2\pi}{p} = n
\]

\[
\begin{align*}
    E - c \sin E &= M_0 + \mu (t \cdot t_0) - M, \\
    r \sin \vartheta &= a \sqrt{1 - e^2} \sin E, \\
    r \cos \vartheta &= a (\cos E - e), \\
    u &= \vartheta + \omega
\end{align*}
\]

(2) (3)

The first three equations (set 1) give the position of the orbiting body in a rectangular coordinate system. The \(X\) axis is toward the Vernal Equinox and the \(Z\) axis is in the direction from the origin toward the celestial pole or the Ecliptic pole (depending on the system of the orbital elements). The coordinate system is righthanded.

The orbital elements, constants and various intermediate quantities used in the equations are listed below.

The orbital elements are:

- \(a\) = semi major axis (kilometers)
- \(i\) = inclination of orbit to ecliptic or equator (Radians)
- \(T\) = time of pericenter passage (Julian Date)
- \(e\) = eccentricity
- \(\omega\) = the argument of pericenter (the angle between the ascending node and pericenter measured along the orbit) (Radians)
\[ \Omega = \text{the longitude of the ascending node (the angle between the Vernal Equinox and the line of nodes - measured along the ecliptic or equator from the Vernal Equinox to the ascending node) (Radians)} \]

and \( P = \text{the period in years.} \)

Other quantities used in the equations:

\[ m = \text{mass of orbiting body} \]

\[ \mu = n = \text{the mean angular motion} \]

\[ k = \text{gaussian constant} = \frac{2 \pi a^{3/2}}{P \sqrt{1 + e}} \]

\[ \pi = 3.14159265 \]

\[ E = \text{eccentric anomaly} \]

\[ M = \text{mean anomaly} \]

\[ M_0 = \text{mean anomaly at time } t_0 \]

\[ t_0 = T = \text{time when } M_0 = 0 \]

\[ v = \text{true anomaly (the angle between pericenter and the position of the body – measured along the orbit)} \]

\[ r = \text{distance from primary to the orbiting body} \]

\[ u = \text{longitude of pericenter (sometimes designated } \theta \text{) – the sum of the angles } v \text{ and } \omega \]

The equations in set (1) were differentiated with respect to time to obtain the following set of equations defining the velocity vector of the body:

\[ V_x = \left( \frac{\mu}{V_3} \right) \left[ aV_2(C_4C_3C_7 - C_5C_8) - C_1 V_1(C_5C_7 + C_4C_3C_8) \right] \]

\[ V_y = \left( \frac{\mu}{V_3} \right) \left[ C_1 V_1(C_4C_5C_6 - C_3C_7) - aV_2(C_4C_5C_7 + C_3C_8) \right] \]

\[ V_z = \left( \frac{\mu C_2}{V_3} \right) (C_1C_3V_1 - aC_7V_2), \]

where

\[ C_1 = a (1-e^2)^{1/2} \]

\[ C_2 = \sin (i) \]
\[ C_3 = \sin (\Omega) \]
\[ C_4 = \cos (i) \]
\[ C_5 = \cos (\Omega) \]
\[ C_7 = \sin (\omega) \]
\[ C_8 = \cos (\omega) \]
\[ V_1 = \cos (E) \]
\[ V_2 = \sin (E) \]
\[ V_3 = \left[ 1 - e \cos (E) \right] \]

The program VELSAT uses this set of equations in this form with orbital elements referred to the equatorial system. VELSUN uses the orbital elements of the Earth referred to the ecliptic system which leads to a simplified set of equations since in this case \( i = 0 \) and \( \Omega \) can be given an arbitrary value (set \( \Omega = 0 \)). The following set of equations, derived from set (4) give the rectangular velocity components of the Earth in the ecliptic system:

\[
\begin{align*}
V'_x &= -\left( \frac{\mu}{V_3} \right) (aC_8V_2 + C_1C_7V_1) \\
V'_y &= \left( \frac{\mu}{V_3} \right) (C_1C_8V_1 - aC_7V_2) \\
V'_z &= 0.0
\end{align*}
\]

To obtain the velocities in the equatorial system from these the following relations are used:

\[
\begin{align*}
V_x &= V'_x \\
V_y &= V'_y \cos (\epsilon) \\
V_z &= V'_y \sin (\epsilon),
\end{align*}
\]

where \( \epsilon \) is the obliquity (about \( 23^\circ \)).
Since these equations involve the eccentric anomaly, \( E \), in addition to the orbital elements it was necessary to solve equation (2) (Kepler's Equation) for \( E \) where the known values are \( e, M_0, \mu, t_0 \) and \( t \). In order to solve this equation the following series approximation (W. M. Smart, 1965) was used:

\[
E = M + \left( e - \frac{e^3}{8} \right) \sin(M) + \left( \frac{1}{2} \right) e^2 \sin(2M) + \left( \frac{3}{6} \right) e^3 \sin(3M)
\]

The eccentricity of the IUE's orbit is 0.23 which causes an error in \( E \) with this approximation of about 0.1 radian. The Earth's orbital eccentricity is so small that the approximation causes no appreciable error.

**FORTRAN LISTINGS**

Listings of the two FORTRAN subroutines, VELSUN and VELSAT are given in Appendix A. Program documentation is in the form of in-line comment statements. Table 1 presents a set of sample input and output for each of the subroutines.

C. Harvel

References:


SUBROUTINE VELSUNT(TIME,VX,VY,VZ,IERERROR)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 INC,NODE,M0

ORIGINAL PROGRAM WRITTEN BY HOWARD L. COHEN AND ARTHUR
YOUNG, INDIANA UNIVERSITY ASTRONOMY DEPT. (PGM NAME
WAS 'ORBVAL'). CIRCA 1965

PROGRAM REWRITTEN BY C. HARVEL 1/24/78
COMPUTER SCIENCE CORPORATION, DEPT. OF ASTRONOMY

PROGRAM MODIFIED BY DAK 1/20/78
GIVEN THE TIME THIS PROGRAM COMPUTES THE THREE
COMPONENTS OF THE EARTH'S RADIAL VELOCITY, VX,VY,VZ(RIGHT
HANDED COORDINATE SYSTEM---X POSITIVE TOWARD THE VERNAL
EQUINOX AND Z POSITIVE TOWARD THE NORTH.
VX,VY,VZ IN KM/SEC, TIME=JD-2400000.0D0( ELEMENTS ARE IN
ECLIPTIC SYSTEM.) THE ERROR CODE IERROR=0 IF THERE ARE
NO ERRORS AND 1 IF THE TIME GIVEN IS OUTSIDE THE
RANGE JD1 TO JDF.

REAL*8 MA,MU,JDI,JDF,E(3),OB(3),OM(3),M(3),SP(2)

DATA PIE/3.141592653589793/
DATA JDI,JDF /1.5024D0, 5.15445D0/
DATA E /9.18751D-4, 1.14440-9, 9.4017/
DATA OB /8.9939319747D0, 6.2190090-9, 2.146755D-17/
DATA OM /1.766368070D0, 8.214999-7, 5.9166660-15/
DATA M /6.265683700D0, 1.720196977D-2, 1.95476880D-15/
DATA SP /366.25636042D0, 1.1017/
DATA A/1.49598/
IERROR=0

THE DATA ABOVE ARE TAKEN FROM THE AMERICAN EPHEMERIS
AND NAUTICAL ALMANAC(1980) PAGE 544 (THE EXPLANATION
SECTION.). JDI=THE EPOCH OF THE DATA(JULIAN DATE OF
EPOCH MINUS 2400000), JDF=FINAL JULIAN DATE BEYOND
WHICH THESE DATA ARE INVALID. E=THE ECCENTRICITY(2ND
AND 3RD ENTRIES IN THESE ARRAYS ARE THE 1ST AND 2ND TIME
DERIVATIVES ), OB=THE UBLIQUITY OF THE ECLIPTIC(IN RADIANS),
OM=THE LONGITUDE OF PERIHELION(RADIANS). M=THE MEAN ANOMALY
(RADIANS), SP=THE SIDERIAL PERIOD(DAYS) AND A=THE SEMIMAJOR
AXIS(KILOMETERS).

IF(TIME.LT.JD1).OR.(TIME.GT.JDF) GO TO 5
GO TO 10
5 IERROR=1
RETURN
STOP

10 D= TIME-JDI
D2= D*D
C CALCULATE THE ECCENTRICITY
ECC= E(1) - E(2) * D - E(3) * D2
C CALCULATE THE UBLIQUITY OF THE ECLIPTIC
OB= OB(1) - OB(2) * D - OB(3) * D2
C CALCULATE THE LONGITUDE OF PERIHELION
UMEGA= UM(1) + UM(2) * D + UM(3) * D2
C CALCULATE THE MEAN ANOMALY
MA = M(1) + M(2) * D - M(3) * D2
C
CALCULATE THE ECCENTRIC ANOMALY
EA = MA + ECC*(ECC**3/6.08)*DSIN(MA) + (0.5*ECC**2 - DSIN(2.0*MA) + (0.375*(ECC**2)*DSIN(3.0*MA)))
C
CALCULATE THE EARTH'S SIDERIAL PERIOD
P = (SP(1) + SP(2) * D)*8.64D0/4
C
CALCULATE THE MEAN MOTION
MU = (2.0*PIE)/P
C
C
C1 = A*(1.0 - ECC**2)**0.5
C
C2 THROUGH C6 NOT USED FOR EARTH ORBIT
C = DSIN(OMEGA)
C0 = DCOS(OMEGA)
V1 = DCOS(EA)
V2 = DSIN(EA)
V3 = (1.0 - ECC*V1)
C
VXACL = -(MU/V3)*((A*C0*V2 + C1*C7*V1)
VYACL = (MU/V3)*((C1*C9*V1 - A*C7*V2)
VZACL = 0.008
C
VXACL, VYACL AND VZACL ARE THE VELOCITIES IN THE ECLIPTIC SYSTEM.
C
VX = VXACL
VY = VYACL*DCOS(OBL)
VZ = VYACL*DSIN(OBL)
C
RETURN
C
C
END
SUBROUTINE VELSAT(TIME,VX,VY,VZ,IERRO)
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 INC,MA,PI,PIE/4,21632D0,
0.493454104,4.4199508,3.2396930D0,4.7203308D0,3.3952750D0,
DATA JD1,JDF / 43199.0,46200.0 / ,
THE DATA ABOVE ARE TAKEN FROM THE IUE PREDICTED
SATELLITE MAP TABLE (NASA, GODDARD--NOV. 22, 1979).
ALL DATA ARE FOR THE EPOCH JD2444199.5 (NOV. 22, 1979).
A=SEM. MAJOR AXIS (KM), INC=INCLINATION TO EQUATOR (RADIANS),
T=EPOCH, ECC=ECCENTRICITY, OMEGA=LON. OF PERIGEE
(RADIANS), NODE=R. ASCENSION OF THE ASCENDING NODE
(RADIANS), P=SIDERIAL PREIOD, MB=MEAN ANOMALY AT EPOCH.

IERRO=0
IF(TIME.LT.JD1.OR.TIME.GT.JDF) GO TO 5
GO TO 10
5 IERRO=1
RETURN
STOP
C
10 MA=MA+(TIME-T)*8.64D0+MB
EA=MA+(ECC**2)/8.1*DSIN(PIE)+(8.5*ECC*ECC*
1 DSIN(2.0*MA)+(0.375*(ECC**3)*DSIN(3.0*MA)))
C
C1=A*(1.0-ECC**2)**2*0.5
C2=DSIN(INC)
C3=DSIN(NODE)
C4=DCOS(INC)
C5=DCOS(NODE)
C6= NOT USED
C7=DSIN(OMEGA)
C8=DCOS(OMEGA)
TABLE 1
Sample Input/Output Values for Programs

<table>
<thead>
<tr>
<th>Program</th>
<th>Input Time JD-2400000</th>
<th>Vx (km/sec)</th>
<th>Vy (km/sec)</th>
<th>Vz (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VELSUN</td>
<td>43251.0D0</td>
<td>0.13207D+02</td>
<td>-0.24371D+02</td>
<td>-0.10568D+02</td>
</tr>
<tr>
<td>VELSAT</td>
<td>43251.0D0</td>
<td>0.18857D+01</td>
<td>0.15146D+01</td>
<td>-0.54586D+00</td>
</tr>
</tbody>
</table>