Tuesday 15th October, 2013
Preprint typeset using LaTeX style emulateapj v. 11/10/09

HUBBLE FRONTIER FIELDS: STRONG GRAVITATIONAL LENSING PRIMER

Dan Coe[1]

Tuesday 15th October, 2013

ABSTRACT

By combining the powers of the Hubble Space Telescope and gravitational lensing by galaxy clusters, the Hubble Frontier Fields (HFF) will obtain the deepest images of our universe to date. The lensed galaxies revealed will include some of the most distant yet known. Some, but not all, of their observed and derived properties will be affected by strong gravitational lensing. Lens models of the six HFF clusters have been produced by five teams for distribution to the community via MAST. To facilitate community use of those models, this short document explains the basics of strong gravitational lensing due to galaxy clusters.

Subject headings: gravitational lensing: strong – galaxies: clusters – cosmology: dark matter — galaxies: high redshift

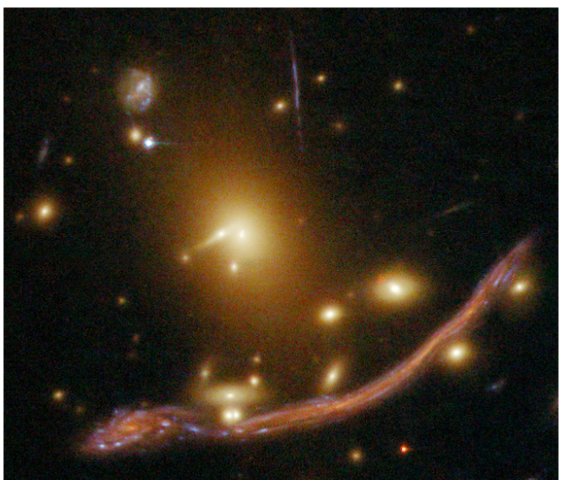

Figure 1. Examples of strong gravitational lensing observed in HST ACS/WFC imaging of Abell 370 obtained prior to the Frontier Fields. Most prominently, the bright red-blue arc at bottom snakes between yellower cluster galaxies which deflect its light. Credit: NASA, ESA, the Hubble SM4 ERO Team, and ST-ECF.

1. INTRODUCTION

This primer is written for the Hubble Frontier Fields (HFF) Director's Discretionary Time (DDT) program which is observing six massive galaxy clusters with the *Hubble Space Telescope*. These images will reveal strong gravitational lensing of background galaxies due to the cluster mass (e.g., Fig. 1). By analyzing this lensing, we can derive the cluster mass distribution to a degree of accuracy which is being quantified in detail by ongoing research. For a recent review of cluster lensing, see Kneib & Natarajan (2011).

This primer is intended to facilitate use of lens models distributed by the HFF lens modelers to the community via the Mikulski Archive for Space Telesopes (MAST).[2] Below we briefly describe the effects of strong gravitational lensing on observables (§2) and the equations that govern strong lensing (§3).

[1] Space Telescope Science Institute, Baltimore, MD, USA; DCoe@STScI.edu

[2] http://archive.stsci.edu/prepds/frontier/lensmodels/

2. OBSERVABLES AND DERIVED QUANTITIES AFFECTED BY LENSING

Table 1 summarizes the degree to which various observables are affected by gravitational lensing.

Lensing conserves surface brightness. Magnified lensed galaxies appear larger and thus brighter. Observed luminosities and derived quantities must be corrected for lensing magnifications, inheriting their full uncertainties from the lens modeling. These derived quantities include star formation rate and stellar mass. However, the ratio of these two quantities, specific star formation rate (sSFR), is not affected by lensing because both quantities are affected equally by the luminosity magnification.

Lensing is achromatic (does not affect galaxy colors). So quantities derived from colors or spectra such as redshift, age, metallicity, extinction, and rest-frame UV slope (β) do not need to be corrected for lensing. (Second-order uncertainties may be introduced for derived quantities that depend on a luminosity prior.)

Quantities that depend on integrating over the lens model area are less susceptible to model uncertainties than local magnification estimates. Such derived quantities include lensed number counts, luminosity functions, and star formation rate densities. See upcoming work for the expected uncertainties in these quantities.

3. LENS EQUATIONS

Galaxies may be strongly lensed into multiple observed images and/or long arcs. As a shorthand, we often refer to all of these as "arcs". For comparison, weak lensing (slight stretching or "shear") may be detected statistically by averaging over many background galaxies observed further from the lensing cluster core.

Observed strong lensing deflections are proportional to, and thus constrain, the projected mass along the line of sight between us and the lensed galaxy. In the thin lens approximation, all of this mass is contained in the lens, or in our case, the galaxy cluster. The deflections also scale geometrically with the redshifts of the lens and the lensed galaxy (the "source" to distinguish it from its observed lensed image) as described below.

A massive body with projected mass surface density $\kappa(\vec{\theta})$ deflects light around it by an angle $\vec{\alpha}(\vec{\theta}) = \vec{\theta} - \vec{\beta}$

Table 1
Observables and Derived Quantities Affected by Gravitational Lensing

Full uncertainties	Reduced uncertainties	No uncertainties introduced
Luminosity: - Star Formation Rate - Stellar Mass	(integrated quantities) Number counts: - Luminosity function - Star formation rate density	Colors: - Redshift - Age - Metallicity - Extinction - Rest-frame UV slope (β) Specific star formation rate (SFR / mass)

from its true position on the sky $\vec{\beta}$ to its observed position $\vec{\theta}$ (Fig. 2):

$$\nabla \cdot \vec{\alpha} = 2\kappa. \tag{1}$$

The surface density $\kappa = \Sigma/\Sigma_{crit}$ is defined in units of the lensing critical density at the redshift of the lens. The critical density is that generally required for strongly lensed multiple images to be produced. It is a function of source redshift as given by:

$$\Sigma_{crit} = \frac{c^2}{4\pi G}\frac{D_S}{D_L D_{LS}}, \tag{2}$$

involving a ratio of the angular-diameter distances from observer to source $D_S = D_A(0, z_S)$, observer to lens $D_L = D_A(0, z_L)$, and lens to source $D_{LS} = D_A(z_L, z_S)$. For a flat universe ($\Omega = \Omega_m + \Omega_\Lambda = 1$), angular-diameter distances are calculated as follows (Fukugita et al. 1992, filled beam approximation; see also Hogg 1999 and online calculators such as iCosmos[3]):

$$D_A(z_1, z_2) = \frac{c}{1+z_2}\int_{z_1}^{z_2}\frac{dz\prime}{H(z\prime)} \tag{3}$$

$$= D_A(0, z_2) - \left(\frac{1+z1}{1+z2}\right) D_A(0, z_1) \tag{4}$$

where (again for a flat universe) the Hubble parameter varies with redshift as:

$$H(z) = H_o\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}. \tag{5}$$

Thus the critical density Σ_{crit} is a function of the source redshift. This follows because the deflection angle $\vec{\alpha}$ is a function of source redshift. As source redshift decreases, the light bend angle $\tilde{\alpha}$ remains constant, which (imagine moving the galaxy in Fig. 2 inward along the top blue arrow) requires the image deflection to decrease by the distance ratio plotted in Fig. 3:

$$\vec{\alpha} \propto \frac{D_{LS}}{D_S}. \tag{6}$$

The normalized mass surface density also scales with this ratio:

$$\kappa \propto \frac{D_{LS}}{D_S} \tag{7}$$

[3] http://www.icosmos.co.uk

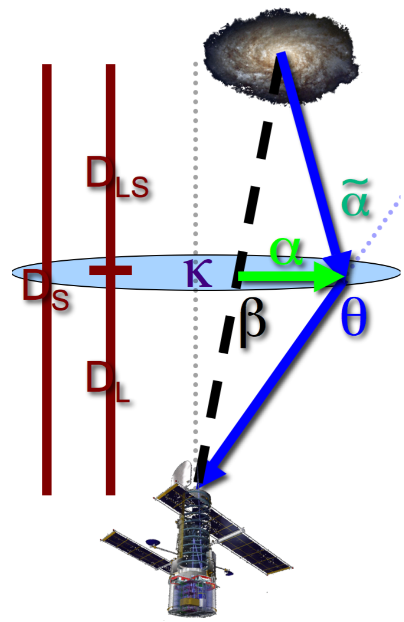

Figure 2. A lens with mass distribution κ deflects the light from a background galaxy by an angle $\tilde{\alpha}$. In the absence of the lens, the background galaxy would appear at its true, or "source", position $\vec{\beta}$. The intervening mass deflects its light by an amount $\vec{\alpha}$ to position $\vec{\theta}$. The deflection angle α on our sky is related to the actual bend angle $\tilde{\alpha}$ of the light ray via $\alpha D_S = \tilde{\alpha} D_{LS}$. The distances D_S, D_{LS}, and D_L are measured as angular diameter distances. Reprinted from Coe et al. (2008).

as does the weak lensing shear:

$$\gamma \propto \frac{D_{LS}}{D_S}. \tag{8}$$

These distance ratios are plotted in Fig. 3 for lenses with redshifts between 0.2 and 0.9.

Magnifications μ do not scale linearly with this ratio, but rather as:

$$1/\mu = (1-\kappa)^2 - \gamma^2 \tag{9}$$

That is, each quantity κ and γ must be scaled to the appropriate redshift before calculating magnifications according to Equation 9. Lensing "critical curves", or regions of high magnification, basically move outward to larger radius as the lensed source redshift increases.

Magnifications can take positive and negative values. A negative magnification merely indicates a flipped mirror image (or "parity") with respect to the lensed source.

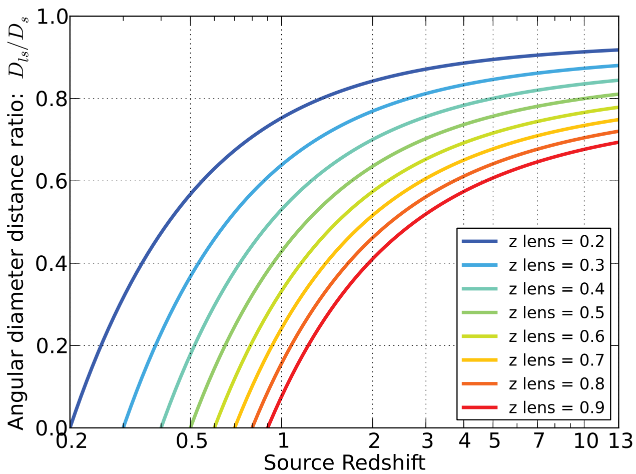

Figure 3. Angular diameter distance ratio D_{LS}/D_S as a function of lens redshift and source redshift. We assumed a flat concordance cosmology with $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.

The lensed galaxy appears both larger (in area) and brighter (in flux) by the magnification factor μ. (Surface brightness is conserved by lensing.) Thus the observed magnitude = the intrinsic magnitude $- 2.5 \log_{10} \mu$.

Faint demagnified galaxy images may also be observed near the cluster core. These smaller images are generally more difficult to detect against the BCG (brightest cluster galaxy) light.

The Hubble Frontier Fields is a Space Telescope Science Institute (STScI) Director's Discretionary Time (DDT) program of observations with the NASA/ESA Hubble Space Telescope's Advanced Camera for Surveys (ACS) and Wide Field Camera 3 (WFC3). As part of this program, five teams were funded teams to produce gravitational lensing models for distribution to the community via the Mikulski Archive for Space Telescopes (MAST). STScI is operated by the Association of Universities for Research in Astronomy, Inc. under NASA contract NAS 5-26555. ACS was developed under NASA contract NAS 5-32864.

Facilities: HST (WFC3, ACS)

REFERENCES

Coe, D., Fuselier, E., Benítez, N., Broadhurst, T., Frye, B., & Ford, H. 2008, ApJ, 681, 814 [ADS]

Fukugita, M., Futamase, T., Kasai, M., & Turner, E. L. 1992, ApJ, 393, 3 [ADS]

Hogg, D. W. 1999, ArXiv Astrophysics e-prints, astro-ph/9905116 [ADS]

Kneib, J.-P. & Natarajan, P. 2011, A&A Rev., 19, 47 [ADS]