HOW THE ADDITIONAL NOISE CAME*

IUE users of spectra obtained in the high resolution mode have noticed a substantial increase of the noise in the data obtained with the new software, in use since November 1981 at GSFC and March 1982 at Vilspa (Turnrose, Thompson and Bohlin, 1982). This fact is usually attributed to the normal increase of noise expected while doubling the sampling. Nevertheless, looking at some IUE high resolution spectra suggests that an additional effect is present. We suspected that the relative position of the pseudo-slit centre with respect to individual pixels may play a role in the new software. A model of the procedure used to obtain spectra from photometrically corrected images has been constructed to study this problem.

I Ideal situation

We suppose that only one order is present, and the signal I at wavelength λ is spread in the x-direction perpendicular to the order following a gaussian law,

$$S_\lambda(x) = \frac{I(\lambda)}{\sqrt{2\pi}\,\sigma} \exp\left(-(x-x_o)^2/2\sigma^2\right) \qquad (1)$$

where x_o is the position of the centre of the order and σ the order width.

If this signal is collected with a receptor of sufficient length in the x-direction and with constant efficiency E, one should measure

$$\frac{I(\lambda)\;E}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} \exp\left(-(x-x_o)^2/2\sigma^2\right) dx = I(\lambda)\;E \qquad (2)$$

II Extraction with an infinite row of pixels

Suppose now that the extraction is carried out with a detector made of a set of square pixels, the diagonals of which lie in the x-direction. The response of such a receptor is proportional to its surface area and the response $R_j(x)$ of an individual pixel located between abscissae j and j+1 can be represented as follows:-

*Reprinted from IUE ESA Newsletter No. 18, December, 1983, pp. 81-86

- null outside [j,j+1] (3)
- $R_j(x) = P(x)\ E = 4\ (x-j)\ E$ for $j < x < j+1/2$
- $R_j(x) = P(x)\ E = 4\ (j+1-x)\ E$ for $j+1/2 < x < j+1$

The length here and below is the pixel-diagonal and the factor 4 is chosen to normalise the mean value of $R(x)$ to E. The contribution C_j of a single pixel may be evaluated analytically:

$$C_j = \frac{4\ I(\lambda) E}{\sqrt{2\pi}} \int_j^{j+1/2} (x-j)\ \exp\ (-(x-x_o)^2/2\sigma^2)\ dx \qquad (4)$$

$$\int_{j+1/2}^{j+1} (j+1-x)\ \exp\ (-(x-x_o)^2/2\sigma^2)\ dx$$

$$C_j = 4I(\lambda)E\left\{\frac{\sigma}{\sqrt{2\pi}}\left[\exp\left(-\frac{(j-x_o)^2}{2\sigma^2}\right) + \exp\left(-\frac{(j+1-x_o)^2}{2\sigma^2}\right) - 2\exp\left(-\frac{(j+1/2-x_o)^2}{2\sigma^2}\right)\right]\right.$$
$$+\frac{1}{2}\left[(j-x_o)\left\{\mathrm{erf}\left(\frac{j-x_o}{\sqrt{2}\sigma}\right) - \mathrm{erf}\left(\frac{j+1/2-x_o}{\sqrt{2}\ \sigma}\right)\right\}\right.$$
$$\left.\left.(j+1-x_o)\left\{\mathrm{erf}\left(\frac{j+1-x_o}{2}\right) - \mathrm{erf}\left(\frac{j+1/2-x_o}{2}\right)\right\}\right]\right\} \qquad (5)$$

where erf is the usual statistical error function.

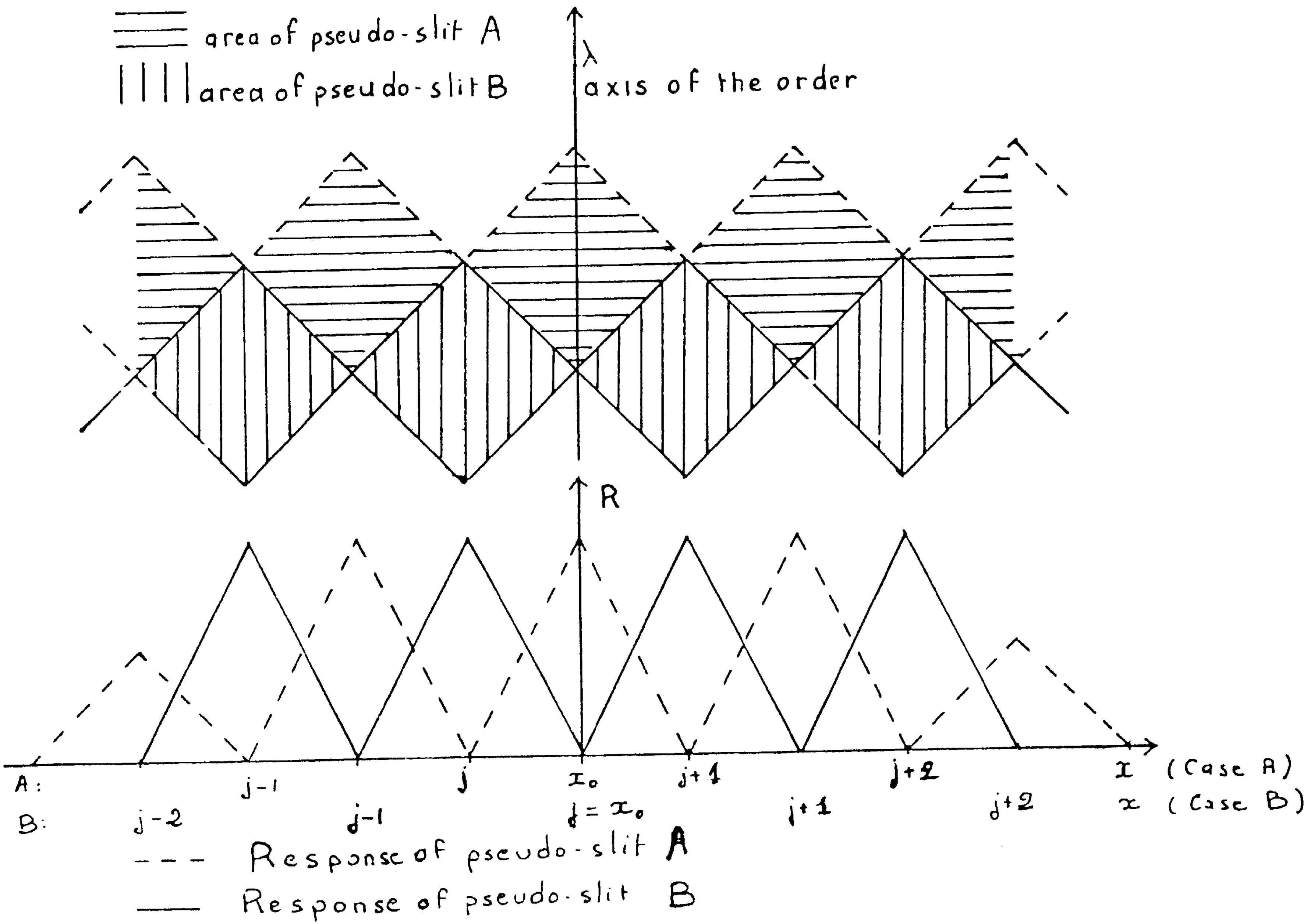
Although the mean value of R_j over [j,j+1] is E a hypothetical sum over all C_j will not lead to the value $I(\lambda)E$, except in the ideal situation.

The table below, showing the results of summation over all C_j of significant value, illustrates the cases when the axis of the order crosses a pixel right in the middle (case A, $x_o = j+1/2$), or when it is located exactly between two pixels (case B, $x_o = j$, see figure). For the sake of simplicity, it will be assumued that $I(\lambda)E=1$. The adopted values of are suggested following De Boer, Preussner & Grewing (1983) and by personal experience.

<u>Summation of an infinite row</u>

σ	0.6	0.8	1.0	1.2	1.4
Case A:	1.000665	1.000003	1.000000	1.000000	1.000000
Case B:	0.999335	0.999997	1.000000	1.000000	1.000000

No noticeable bias is introduced in this case.


area of pseudo-slit A
area of pseudo-slit B
λ
axis of the order
R
A:
j-1
j
x₀
j+1
j+2
x (Case A)
B:
j-2
j-1
j = x₀
j+1
j+2
x (Case B)
Response of pseudo-slit A
Response of pseudo-slit B

III Extraction with a finite pseudo-slit exactly centred on the order

We suppose now that we limit the summation to a width of 4 pixel-diagonals as in the new IUE software. If the centre of the pseudo-slit lies exactly between two pixels (case B), the extraction consists of a simple sum of the contribution of 4 pixels: otherwise, the edge of the pseudo-slit comes across the two extreme pixels. In this latter case, the contribution of those pixels is affected in the summation by a factor which is the ratio of the area within the slit to the total area of the pixel. By this means the pseudo-slit keeps a virtual area of 4 pixels. The results of a summation with a pseudo-slit, under the assumption that the actual order centre coincides exactly with the pseudo slit centre are shown below for the two extreme cases mentioned above.

Summation with a pseudo-slit (width = 4)

σ	0.6	0.8	1.0	1.2	1.4
Case A:	0.9964	0.9730	0.9319	0.8805	0.8251
Case B:	0.9998	0.9900	0.9589	0.9092	0.8512

Of course there is a general loss of signal as is already known, and taken into account in the interpretation of the results. But there is also a significant difference between cases A and B.

Unfortunately, these two extremes are routinely encountered with the extraction of the new IUE software in two successive points of the "gross spectrum". In fact, the relative position of the slit centre versus pixel centre differs in two successive pseudo-slits by approximately 0.5 pixel-diagonals (see figure), which is the worst possible case. Therefore the new software introduces a systematic mathematical bias that can be as high as 3% of the Flux Number between Case A and Case B. This bias acting systematically in opposite directions on two successive points of the extracted spectra, appears as noise in the plotted results.

It must be stressed here that using a pseudo-slit with its edge falling across a pixel does not mean that we are able to distinguish between photons coming inside or outside the slit. It only means that we apply a correction factor to the contribution of the whole pixel proportional to the area of the pixel within the pseudo-slit. This pixel-slicing operation supposes that the pixel is more or less uniformly exposed, which is not the case in an IUE image with flux varying rapidly in the x-direction. This procedure rejects too many photons in the extreme pixels of the pseudo-slit.

IV Extraction with a pseudo-slit not exactly centred on the order

The table below illustrates the case when the actual postion of the centre of the order differs from the position of the centre of the pseudo-slit (generally the theoretical position of the order). When the distance between the two centres is 0.3 pixel-diagonals, the following results are obtained:-

Summation with a de-centred pseudo-slit

σ	0.6	0.8	1.0	1.2	1.4
Case A:	0.9907	0.9635	0.9212	0.8703	0.8162
Case B:	0.9987	0.9843	0.9493	0.8988	0.8418

From the point of view of the systematic mathematical "noise" introduced, there is no qualitative difference. However, the loss of signal that can result from incorrectly positioning the centre of the pseudo-slit is obvious. As the theoretical position of an order may differ from the actual position by as much as 0.5 pixel-diagonals (Bohlin & Turnrose, 1982), this effect is worth noting.

V How do the old and new software differ?

In the old software, the pseudo-slit shape did not vary from one extraction point to the next, the extreme pixels were always wholly taken into account, and the effect mentioned above could not appear. In order to keep the shape of the pseudo-slit unchanged, the old software allowed the slit centre to deviate slightly from the exact position of the order. Consequently, there was a drift of the slit centre with respect to the theoretical position of the order as the extraction progressed along that order.

Compensation for this drift in x_0 was made by means of discrete jumps in the slit position. This procedure is expected to add some small oscillations to the global sensitivity of IUE, but such a study is beyond the scope of this paper.

VI What can be done?

The most straightforward manner to eliminate the effect introduced by the new software is to add three successive λ-points of extracted spectra with respective weights 1/4, 1/2, 1/4. This should simulate fairly well a receptor with a flat response in the x-direction. However, this causes drastic degradation in the λ-resolution. Another way is to adjust $I(\lambda)$ with a non-linear least-square minimisation procedure along the x-direction, taking into account the particular geometry of the problem. Work is under way to provide a suitable extraction procedure starting from the photometrically corrected image (2nd file).

I would like to thank here Dr A. Talavera and C. Catala for fruitful discussions and Dr F. Praderie for comments on the manuscript.

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