IUE DATA REDUCTION

XVI. Orbital Velocity Corrections

INTRODUCTION

The resolving power (R =λ/FWHM) of the IUE spectrographs varies from 1.0 X 10^4 to 1.5 X 10^4 over the entire wavelength range covered (Boggess, et al., 1978). Therefore the IUE velocity resolution varies from 20 km s^{-1} to 30 km s^{-1}. If it is assumed that the centroid of a line can be determined to approximately 10% of its FWHM, then a measured radial velocity should be accurate to about 2 km s^{-1} (best case). The orbital velocity of the Earth about the Sun (~30 km s^{-1}) and the velocity of the spacecraft about the Earth (~4 km s^{-1} at perigee) are both larger than the best possible velocity determinations and their effect should be removed from the data.

Two subprograms, VELSUN and VELSAT, have been written for IUESIPS, the International Ultraviolet Explorer Spectral Image Processing System, to calculate orbital velocities. VELSUN determines the velocity vector of the Earth at a given time using the orbital elements of the Earth and the time derivatives of these elements (A.E., 1980). VELSAT determines the velocity vector of the spacecraft about the Earth at a given time using all the orbital elements of the spacecraft for Nov. 22, 1979, except for the period, which is set to exactly one sidereal day. Since the orbit of the spacecraft is periodically adjusted to maintain a sidereal period and, moreover, an approximately fixed ground path, it is not necessary to update the orbital elements used by the program. This program is accurate to ±0.25 km s^{-1} over the entire life of the spacecraft (launch to present), and VELSUN is accurate to better than 0.01 km s. Both of these subprograms will be added to IUESIPS in the near future.

EQUATIONS USED BY VELSUN AND VELSAT

The following sets of equations giving the rectangular coordinates of a body in its orbit as a function of the orbital elements were adopted (A. D. Dubyago, 1961):

$$\left.\begin{aligned} x &= r(\cos u \cos\Omega - \sin u \cos i \sin\Omega), \\ y &= r(\cos u \sin\Omega + \sin u \cos i \cos\Omega), \\ z &= r \sin u \sin i. \end{aligned}\right\} \quad (1)$$

and

$$\mu = \frac{k\sqrt{1+m}}{a^{3/2}} = \frac{2\pi}{P} = n$$

$$E - e \sin E = M_0 + \mu(t - t_0) = M, \quad (2)$$

$$\left.\begin{aligned} r \sin v &= a\sqrt{1 - e^2} \sin E, \\ r \cos v &= a(\cos E - e), \\ u &= v + \omega \end{aligned}\right\} \quad (3)$$

The first three equations (set 1) give the position of the orbiting body in a rectangular coordinate system. The +X axis is toward the Vernal Equinox and the +Z axis is in the direction from the origin toward the celestial pole or the Ecliptic pole (depending on the system of the orbital elements). The coordinate system is righthanded.

The orbital elements, constants and various intermediate quantities used in the equations are listed below.

The orbital elements are:

a = semi major axis (kilometers)

i = inclination of orbit to ecliptic or equator (Radians)

T = time of pericenter passage (Julian Date)

e = eccentricity

ω = the argument of pericenter (the angle between the ascending node and and pericenter measured along the orbit) (Radians)

Ω = the longitude of the ascending node (the angle between the Vernal Equinox and the line of nodes - measured along the ecliptic or equator from the Vernal Equinox to the ascending node) (Radians)

and P = the period in years.

Other quantities used in the equations:

m = mass of orbiting body

$\mu = n$ = the mean angular motion

k = gaussion constant $= \dfrac{2\pi a^{3/2}}{P\sqrt{1+m}}$

$\pi = 3.14159265$

E = eccentric anomaly

M = mean anomaly

M_o = mean anomaly at time t_o

$t_o = T$ = time when $M_o = 0$

v = true anomaly (the angle between pericenter and the position of the body - measured along the orbit)

r = distance from primary to the orbiting body

u = longitude of pericenter (sometimes designated θ) - the sum of the angles v and ω

The equations in set (1) were differentiated with respect to time to obtain the following set of equations defining the velocity veotor of the body:

$$\left.\begin{aligned} V_x &= (\mu/V_3)\left[aV_2(C_4C_3C_7 - C_5C_8) - C_1V_1(C_5C_7 + C_4C_3C_8)\right] \\ V_y &= (\mu/V_3)\left[C_1V_1(C_4C_5C_8 - C_3C_7) - aV_2(C_4C_5C_7 + C_3C_8)\right] \\ V_z &= (\mu C_2/V_3)\ (C_1C_8V_1 - aC_7V_2)\ , \end{aligned}\right\} \qquad (4)$$

where

$C_1 = a\ (1-e^2)^{1/2}$

$C_2 = \sin\ (i)$

$C_3 = \sin(\Omega)$

$C_4 = \cos(i)$

$C_5 = \cos(\Omega)$

$C_7 = \sin(\omega)$

$C_8 = \cos(\omega)$

$V_1 = \cos(E)$

$V_2 = \sin(E)$

$V_3 = \left[1 - e\cos(E)\right]$

The program VELSAT uses this set of equations in this form with orbital elements referred to the equatorial system. VELSUN uses the orbital elements of the Earth referred to the ecliptic system which leads to a simplified set of equations since in this case $i = 0$ and Ω can be given an arbitrary value (set $\Omega = 0$). The following set of equations, derived from set (4) give the rectangular velocity components of the Earth in the ecliptic system:

$$\left.\begin{aligned} V_x' &= -(\mu/V_3)\,(aC_8V_2 + C_1C_7V_1) \\ V_y' &= (\mu/V_3)\,(C_1C_8V_1 - aC_7V_2) \\ V_z' &= 0.0 \end{aligned}\right\} \quad (5)$$

To obtain the velocities in the equatorial system from these the following relations are used:

$$V_x = V_x'$$

$$V_y = V_y' \cos(\epsilon)$$

$$V_z = V_y' \sin(\epsilon),$$

where ϵ = the obliquity (about 23°).

Since these equations involve the eccentric anomaly, E, in addition to the orbital elements it was necessary to solve equation (2) (Keplers Equation) for E where the known values are e, M_0, μ , t_0 and t. In order to solve this equation the following series approximation (W. M. Smart, 1965) was used:

$$E = M + (e - \frac{e^3}{8}) \sin(M) + (1/2)e^2 \sin(2M) + (3/8)e^3 \sin(3M)$$

The eccentricity of the IUE's orbit is 0.23 which causes an error in E with this approximation of about 0.1 radian. The Earth's orbital excentricity is so small that the approximation causes no appreciable error.

FORTRAN LISTINGS

Listings of the two FORTRAN subroutines, VELSUN and VELSAT are given in Appendix A. Program documentation is in the form of in-line comment statements. Table 1 presents a set of sample input and output for each of the subroutines.

C. Harvel

References:

A. E. (1980). The American Ephemeris and Nautical Almanac (U. S. Government Printing Office, Washington), page 544.

Boggess, A. et al. (1978). In-Flight Performance of the IUE, Nature, 275, 7.

A. D. Dubyago (1961). The Determination of Orbits (The Macmillan Company, New York), p. 49

Smart W. M. (1965). Text-Book on Spherical Astronomy (Cambridge University Press, Cambridge), Chap. 5.

APPENDIX A

```
      SUBROUTINE VELSUN(TIME,VX,VY,VZ,IERROR)
      IMPLICIT REAL*8(A-H,O-Z)
C
      REAL*8 INC,NODE,MØ
C
C     ORIGINAL PROGRAM WRITTEN BY HOWARD L. COHEN AND ARTHUR
C     YOUNG INDIANA UNIVERSITY ASTRONOMY DEPT.( PGM NAME
C     WAS 'ORBVAL'). CIRCA 1965
C
C     PROGRAM REWRITTEN BY C. HARVEL 1/24/8Ø
C     COMPUTER SCIENCE CORPORATION, DEPT. OF ASTRONOMY
C
C     PROGRAM MODIFIED BY DAK 1/2Ø/78
C     GIVEN THE TIME THIS PROGRAM COMPUTES THE THREE
C     COMPONENTS OF THE EARTHS RADIAL VELOCITY,VX,VY,VZ(RIGHT
C     HANDED COORDINATE SYSTEM--- X POSITIVE TOWARD THE VERNAL
C     EQUINOX AND  Z  POSITIVE TOWARD THE NORTH.
C     VX,VY,VZ IN KM/SEC, TIME=JD-24ØØØØØ.ØDØ( ELEMENTS ARE IN
C     ECLIPTIC SYSTEM.   )  THE ERROR CODE IERROR=Ø IF THERE ARE
C     NO ERRORS AND  1  IF THE TIME GIVEN IS OUTSIDE THE
C     RANGE JDI TO JDF.
C
      REAL*8 MA,MU,JDI,JDF,E(3),OB(3),OM(3),M(3),SP(2)
C
      DATA PIE/3.141592653589 8DØ/
      DATA JDI,JDF /1.5Ø2D4, 5.15445D4/
      DATA E /Ø.Ø167510ØDØ,1.1444D-9, 9.4D-17/
      DATA OB /Ø.4Ø9319747DØ, 6.2179Ø99D-9, 2.1467550D-17/
      DATA OM /1.7666368Ø7DØ, 8.21499D-7, 5.9166660D-15/
      DATA M /6.25658378DØ, 1.72Ø196977D-2, 1.9547688D-15/
      DATA SP /365.25636Ø42DØ, 1.1D-7/
      DATA A /1.496D8/
      IERROR=Ø
C
C     THE DATA ABOVE ARE TAKEN FROM THE AMERICAN EPHEMERIS
C     AND NAUTICAL ALMANAC(198Ø) PAGE 544( THE EXPLANATION
C     SECTION ).  JDI=THE EPOCH OF THE DATA(JULIAN DATE OF
C     EPOCH MINUS 24ØØØØØ), JDF= FINAL JULIAN DATE BEYOND
C     WHICH THESE DATA ARE INVALID, E=THE ECCENTRICITY(2ND
C     AND 3RD ENTRIES IN THESE ARRAYS ARE THE 1ST AND 2ND TIME
C     DERIVITIVES ), OB=THE OBLIQUITY OF THE ECLIPTIC(IN RADIANS),
C     OM=THE LONGITUDE OF PERIHELION(RADIANS), M=THE MEAN ANOMALY
C     (RADIANS), SP=THE SIDERIAL PERIOD(DAYS) AND A=THE SEMIMAJOR
C     AXIS(KILOMETERS).
C
C
C
      IF(TIME.LT.JDI.OR.TIME.GT.JDF) GO TO 5
      GO TO 1Ø
    5 IERROR=1
      RETURN
      STOP
C
C     CALCULATE ELAPSED TIME SINCE JD 24150 2Ø.Ø
   1Ø D= TIME-JDI
      D2= D*D
C     CALCULATE THE ECCENTRICITY
      ECC= E(1) - E(2) * D - E(3) * D2
C     CALCULATE THE OBLIQUITY OF THE ECLIPTIC
      OBL= OB(1) - OB(2) * D - OB(3) * D2
C     CALCULATE THE LONGITUDE OF PERIHELION
      OMEGA= OM(1) + OM(2) * D +OM(3) * D2
C     CALCULATE THE MEAN ANOMALY
```

```
        MA= M(1) + M(2) * D - M(3) * D2
C       CALCULATE THE ECCENTRIC  ANOMALY
        EA=MA+(ECC-(ECC**3)/8.DØ)*DSIN(MA)+(Ø.5*ECC*ECC*
     1  DSIN(2.*MA)+(Ø.375*(ECC**3)*DSIN(3.*MA)))
C       CALCULATE THE EARTHS SIDERIAL PERIOD
        P=( SP(1) + SP(2) * D )*8.64DØ4
C       CALCULATE THE MEAN MOTION
        MU=(2.DØ*PIE)/P
C
C
C
        C1=A*(1.DØ-ECC**2)**Ø.5
C       C2 THROUGH C6  NOT USED FOR EARTH ORBIT
        C7=DSIN(OMEGA)
        C8=DCOS(OMEGA)
        V1=DCOS(EA)
        V2=DSIN(EA)
        V3=(1.DØ-ECC*V1)
C
C
        VXECL= -(MU/V3)*(A*C8*V2+C1*C7*V1)
        VYECL=  (MU/V3)*(C1*C8*V1-A*C7*V2)
        VZECL  = Ø.ØDØ
C
C       VXECL,VYECL AND VZECL ARE THE VELOCITIES IN THE
C       ECLIPTIC SYSTEM.
C
C
        VX= VXECL
        VY= VYECL*DCOS(OBL)
        VZ= VYECL*DSIN(OBL)
C
C
C
        RETURN
        END
C
```

```
      SUBROUTINE VELSAT(TIME,VX,VY,VZ,IERROR)
      IMPLICIT REAL*8 (A-H,O-Z)
C
      REAL*8 INC,NODE,MA,MU,MØ,JDI,JDF
C
C     PROGRAM WRITTEN BY C. HARVEL 1/24/8Ø
C     COMPUTER SCIENCES CORPORATION,DEPT. OF ASTRONOMY
C
C     GIVEN THE TIME (JULIAN DATE) THIS PROGRAM COMPUTES
C     THE THREE COMPONENTS OF THE SATELLITES (IUE)
C     RADIAL VELOCITY---VX,VY,VZ.  COORDINATE SYSTEM IS
C     RIGHT HANDED WITH  X  POSITIVE TOWARD THE VERNAL
C     EQUINOX, AND  Z  POSITIVE TOWARD THE NORTH.
C     VX,VY,VZ ARE IN KM/SEC, TIME=JULIAN-DATE MINUS
C     24ØØØØØ.Ø .  ALL ELEMENTS ARE REFERED TO THE EARTH'S
C     EQUATOR( NOT THE ECLIPTIC). THE ERROR CODE IERROR=Ø
C     IF THERE ARE NO ERRORS AND  1  IF THE TIME IS NOT
C     IN THE RANGE JDI TO JDF.
C
C
C
C
C     TIME=JD-24ØØØØØ.ØDØ
C     VX,VY,VZ=  VELOCITIES IN KM/SEC
C
C
C
      DATA A,INC,T,ECC,OMEGA,NODE,P,MØ,PIE/4.21632DØ4,
     1 Ø.4934541DØ,44199.5DØ,Ø.2359693DØ,4.7283238DØ,3.385275DØ,
     2 8.61642DØ4,4.3Ø32838Ø4DØ,3.14159265359DØ/
      DATA JDI,JDF / 43199.DØ, 462ØØ.DØ /
C
C     THE DATA ABOVE ARE TAKEN FROM THE IUE PREDICTED
C     SATELLITE MAP TABLE (NASA, GODDARD--NOV. 22, 1979).
C     ALL DATA ARE FOR THE EPOCH JD2444199.5 (NOV. 22,1979).
C     A=SEMI MAJOR AXIS(KM), INC=INCLINATION TO EQUATOR(RADIANS),
C     T=EPOCH, ECC=ECCENTRICITY, OMEGA=LONGITUDE OF PERIGEE
C     (RADIANS), NODE=RIGHT ASCENSION OF THE ASCENDING NODE
C     (RADIANS), P=SIDERIAL PREIOD, MØ=MEAN ANOMALY AT EPOCH.
C
C
C
      IERROR=Ø
C
C
      IF(TIME.LT.JDI.OR.TIME.GT.JDF) GO TO 5
      GO TO 1Ø
    5 IERROR=1
      RETURN
      STOP
C
   1Ø MU=(2.*PIE)/P
      MA=MU*(TIME-T)*8.64DØ4+MØ
      EA=MA+(ECC-(ECC**3)/8.)*DSIN(MA)+(Ø.5*ECC*ECC*
     1 DSIN(2.*MA)+(Ø.375*(ECC**3)*DSIN(3.*MA)))
C
      C1=A*(1.-ECC**2)**Ø.5
      C2=DSIN(INC)
      C3=DSIN(NODE)
      C4=DCOS(INC)
      C5=DCOS(NODE)
C     C6=  NOT USED
      C7=DSIN(OMEGA)
      C8=DCOS(OMEGA)
```

```
      V1=DCOS(EA)
      V2=DSIN(EA)
      V3=(1.-ECC*V1)
C
C
      VX=(MU/V3)*(A*(C4*C3*C7-C5*C8)*V2-C1*(C5*C7+C4*C3*C8)*V1)
      VY=(MU/V3)*(C1*(C4*C5*C8-C3*C7)*V1-A*(C4*C5*C7+C3*C8)*V2)
      VZ= (MU*C2/V3)*(C1*C8*V1-A*C7*V2)
C
C
      RETURN
      END
```

TABLE 1

Sample Input/Output Values for Programs

Program	Input Time JD-2400000	Vx (km/sec)	Vy (km/sec	Vz (km/sec)
VELSUN	43251.0D0	0.13207D+02	-0.24371D+02	-0.10568D+02
VELSAT	43251.0D0	0.18857D+01	0.15146D+01	-0.54586D+00